Electrostriction Phenomena in Superfluid ^4He

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Properties of Superfluid ^4He : Compressibility

β = 1/ Bulk Modulus

$$\beta = -\frac{\Delta V}{V_0}\frac{1}{\Delta p}$$

where Δp = applied pressure

$$\boxed{\text{Work done} = +\frac{1}{2\beta V_0}\Delta V^2}$$

1) Water $\beta_w = 46 \times 10^{-6}$ / atm

2) Superfluid ^4He: **see Fig. 7.17 in William Keller's Book "Helium –3 and Helium –4"**

$\beta_4 = 14 \times 10^{-3}$ / atm $= 300 \times \beta_w$

Example: 4He II, **V_0 = 1 lt**
Applied pressure = 1 atm

$\Delta V/V_o$ = 1.4 % ; **Work required 0.7 Joules**

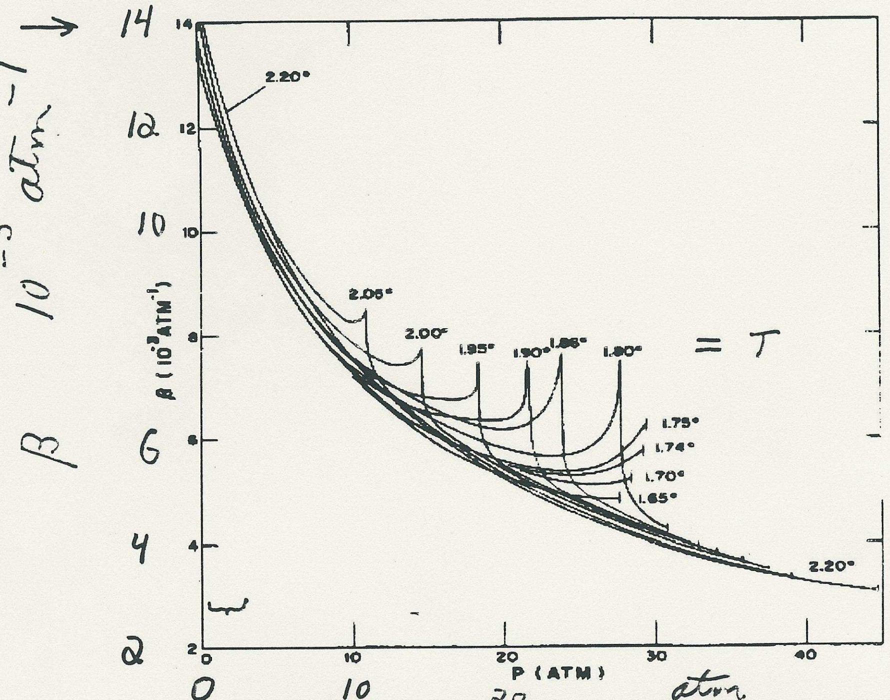


Fig. 7.17. Coefficient of isothermal compressibility for liquid He^4 as a function of pressure. Maxima of cusps indicate crossings of the λ-line [from Grilly (72)].

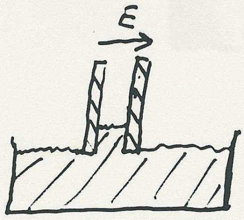
W. Keller, Plenum Press, 1969, p. 256

Properties of Superfluid ^{4}He : Electrostriction

Electromagnetic energy stored in a capacitor:

No dielectric: U_0

With dielectric: U_0 / ε



A. Text Book example

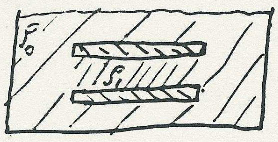
Incompressible fluid dielectric drawn up into capacitor:

Q = CONSTANT

Reduction in EM stored energy
= Increase in gravitational energy

B. Compressible fluid dielectric :

compression of fluid → Increase density → Increase ε



Reduction in EM stored energy
= Work done in compression

Clausius – Mosotti Relation

$$\frac{\epsilon - 1}{\epsilon + 2} = \frac{4\pi \alpha_M}{3M}\rho$$

For $\varepsilon \sim 1$, $\quad \varepsilon \sim 1 + \dfrac{4\pi \alpha_M}{M}\rho$

Example **Superfluid ^{4}He with $\rho_4 = 0.146$ gm/cm^3, $\alpha_M = 0.125$**
$\varepsilon = 1.06$

Energy Balance under compression

FOR AN ISOLATED CAPACITOR,

Let the dielectric constant before compression be: ε_i

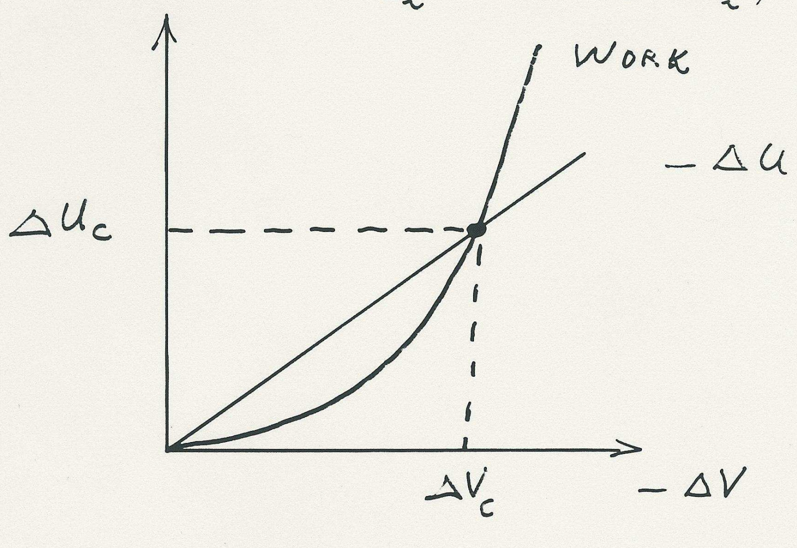
and ΔV be the volume of extra fluid brought into volume V_o

Then:

$$\varepsilon = \epsilon_i - (\epsilon_i - 1)\frac{\Delta V}{V_0}$$

The EM energy shift is

$$\Delta U = -U_i \frac{\Delta \epsilon}{\epsilon_i} = \frac{U_i}{V_0}\left(\frac{\epsilon_i - 1}{\epsilon_i}\right)\Delta V$$



At what value $\Delta V = \Delta V_c$, does

Reduction of EM Energy = work of compression?

$$\boxed{\Delta V_c / V_o = -\beta \left(\frac{U_o}{V_o}\right)\left(\frac{\epsilon_i - 1}{\epsilon_i}\right)\frac{1}{\epsilon_i}}$$

For ^4He $$= -2\left(\frac{14 \times 10^{-3}}{atm}\right)\left(\frac{111\,J}{m^3}\right)\left(\frac{0.05}{1.05}\right)\frac{1}{1.05}$$

$1\,atm = 10^5\,N/m^2$

$$\Delta V_c / V_o = 1.4 \times 10^{-6}$$

For V_o = 1 liter

$$\Delta U_c = 8 \times 10^{-9}\ J$$

This corresponds to a pressure in the superfluid of

$$\Delta p_c = 1.1 \times 10^{-4}\ atm$$

Conclusions

1. He II is relatively compressible.

2. The dielectric constant of He II is relatively insensitive to changes in its density.

3. With electric fields of 50 kV/cm, the energy stored in the compression is small.

4. The equivalent pressure is small.